The Effect of Lever Arm, Distance of Punch Applied, and Mass of Fist on the Force Exerted by a Punch

Thanvir Ahmed and Kishwar Basith

Macomb Mathematics Science Technology Center

Honors Physics

11 A

Mr. McMillan / Mrs. Cybulski / Mrs. Tallman

5 June 2015

The Effect of Lever Arm, Distance of Punch Applied, and Mass of Fist on the Force Exerted by a Punch

Punching is a body movement that is practiced by most individuals whether it is for physical exercise, self-defense, or sport. The purpose of this experiment was to determine which factors influenced the force exerted by a punch the most. The factors that were being tested were the lever arm, the distance at which the punch was being applied from, and the mass of the fist that was punching. The lever arm correlates to the position on the body that would be punched. The hypothesis that was formulated was that if the longest length of the lever arm of 1.4 m is used in combination with the longest distance of punch applied at 0.8 m, and the highest mass of fist of 2.16 kg, then the largest force will be exerted on the punching bag.

This hypothesis was tested to be true. Video analysis was used to measure the angular acceleration and torque. The moment of inertia of the bag was also calculated. These values were then used to calculate the force that was exerted on the punching bag. To analyze this data a Three Factor Design of Experiment (DOE) was conducted.

The results of the DOE concluded that the factor that had the greatest effect upon the force exerted by the punch was the distance at which the punch was applied. The second most significant factor was the lever arm, followed by the mass of the fist, and then the interaction between the lever arm and the distance at which the punch was applied.

Table of Contents

Introduction

Physical confrontation in the form of a brawl is often avoided, but sometimes conflicts are unavoidable, and pacifism is no longer an option. Fighting for sport, such as Mixed Martial Arts (MMA), professional boxing, Ultimate Fighting Championship (UFC), are all growing industries with numerous viewers and endorsements. Whether it is for self-defense or for sport, the human body can be manipulated and used as a tool. One of the most common utilizations of the human body is the punch. This extension of the arm seems simple, but there are numerous factors that can be manipulated in order to maximize the force it exerts. Among these factors are: the lever arm, the distance at which the punch is applied, and the mass of the fist that is punching. The lever arm refers the length along the punching bag where the punch is applied, from the pivot point, the point where the bag is suspended from, to the spot that is being punched. The distance of the punch refers to how far the punch travels to reach the bag. The mass of the fist refers to the mass of the fist used to punch the bag. The mass was manipulated using fighting gloves of different masses.

The purpose of this experiment was to determine which factor or combination of factors of lever arm, distance at which the punch is applied, and the mass of the fist, would have the greatest effect on the force exerted by a punch. Each of these individual factors had a high, standard, and low value. Since there were three different factors, with a high, standard, and low value for each, the data was analyzed by conducting a Three Factor Design of Experiment (DOE).

The lever arm, the distance at which the punch was applied, and the mass of the fist were the three different factors that were being tested. The lever arm can be viewed as the length, or height, of an opponent. This value varies among different fighters, causing variation within the lever arm. A head shot corresponds to the high value for lever arm, while a blow to the legs relates to the low value. The distance at which the punch is applied can vary within real life situations. Professional boxers and MMA fighters often move around to avoid being punched. This causes variation within the distance at which the punch is applied. The mass of the fist varies among different boxers and MMA fighters, with heavyweight boxers, welterweight, etc. These variations were taken into account when determining the different values for the individual factors. Professional fighters often experiment with different values for lever arm and punching distance to maximize the force of their punches. The more force a punch has, the more likely the fighter is to be victorious.

The data that was being measured was the force of each punch. This was accomplished by measuring the angle of displacement and the time created by punching a punching a punching bag, which was recorded on a camera. This angle was measured by utilizing computer software. The torque and angular acceleration were derived from this data. Moment of inertia was constant throughout. The force of the punches was calculated by using all of these values. The force can be derived from these values because torque is equal to the force multiplied by the lever arm, $\tau = Fl\sin(\text{Ø})$, and to angular acceleration multiplied by moment of inertia, $\tau = I\alpha$. Sample calculations are shown in Appendix B.

Review of Literature

A punch is a common utilization of the human body in both professional sports and as a form of self-defense. There are numerous factors that can be manipulated in order to maximize the force of a punch, such as the lever arm, the distance at which the punch is applied, and the mass of the fist that is punching. Force was chosen in this experiment because force is a push or pull upon an object resulting from the object's interaction with another object. Whenever there is an interaction between two objects, there is a force upon each of the objects. The lever arm is the perpendicular distance from the axis of rotation to the line of action of the force. Lever arm is modeled as r or l. If the force is applied at an angle, theta Ø measured in degrees (°), then the lever arm is equal to the radius or length of the object multiplied by the sine of the angle, demonstrated in this equation (Richmond):

$$l = r \times \sin(\text{Ø})$$

The lever arm is measured in meters (m). This is how lever arm was calculated within this experiment. The length of the punching bag from the point of rotation was measured using a meter stick. The angle, or theta, was measured by video recording each trial with an iPhone 6 and then analyzing the video using the Screen Protractor V1.1 Software. The longer the lever arm, the larger the angle of displacement will be (Nave). The distance at which the punch is applied is simply the distance the punch accelerates. This distance is measured in meters (m). The mass of the fist is the mass of the object that is accelerating and exerting the force, this is measured in Kilograms (kg).

Within this experiment, the force exerted on a punching bag was calculated. In order to calculate this force, the moment of inertia of the punching bag was calculated first. Then the Angular acceleration had to be calculated. After both the Angular acceleration and the moment of inertia were calculated the torque was calculated. The lever arm was then calculated. The lever arm and the torque were then used to calculate the force of the punch.

The moment of inertia is a quantity expressing a body's tendency to resist angular acceleration. A point mass, an object rotating about a fixed axis such as a string or chain, has a moment of inertia equal to the mass, m, multiplied by the radius, r, squared (Fitzpatrick).

$$I = \mathrm{m} \times r^2$$

The symbol I is used to represent moment of inertia measured in kilogram-square meters ($kg*m^2$). The point of rotation is the point at which the object rotates about (Fitzpatrick). The punch, a force, was applied to the bag at a length away from the center of mass. The center of mass of the bag was the center of the bag, being a point mass.

The angular acceleration, or rotational acceleration, is the change in angular velocity over time. Angular velocity is the rate of change of angular position of a rotating body (Nave). Angular acceleration is equivalent to the angular displacement divided by the time elapsed squared (Benson).

$$\alpha = \frac{\Delta Ø}{\Delta t^2}$$

The symbol α represents angular acceleration measured in radians per second squared (rad/sec^2). The angular displacement $\Delta\text{Ø}$ is measured in radians. The time elapsed Δt is measured in seconds

The torque of an object is a force that causes rotation. Torque, τ, is equivalent to the moment of inertia, I, of an object multiplied by the angular acceleration, α, (Richmond). This is modeled by the equation shown below.

$$\tau = I\alpha$$

Torque is measured in Newton meters (N*m). Torque is also equal to the force applied multiplied by the lever arm, demonstrated in the equation below (Vawter).

$$\tau = Fl\sin(\text{Ø})$$

The force, measured in Newtons (N), is symbolized as *F* within this equation. The force exerted upon an object can be derived by calculating the torque acting upon that object and by calculating the lever arm of that object. The angle of the lever arm is represented by Ø.This is how the force of each punch was calculated within this experiment.

In preparation for this experiment, past experiments were researched. One experiment was conducted at the National Olympic and Paralympic Academy of Iran by several college students led by Mahdi Cheraghi. The experiment was conducted to analyze the physical strains placed on the body while executing a boxing punch (Cheraghi et al.). Video analysis was used to analyze the body while the subject was throwing a punch. The analysis showed that the neck is constantly in a twisting motion during a punch and there is significant tension present in the spine and waist. These risks were taken into consideration when conducting trials. The research team also concluded that the if the human arm is allowed to extend the maximum distance of the human body, then it will have the largest amount of force. This is because the arm is able to accelerate for the maximum distance. This information supports the hypothesis within this experiment.

Another experiment was conducted by Wayne State University's Biomedical Engineering program, headed by Cindy Bir. The Wayne State research team concluded that a professional boxer could produce a maximum force of 5,000 N. These punches were delivered straight onto a bag with equipment to measure force. It was also found that a sharp blow of 3,300 N has a 25% chance of cracking the ribs of an average person. For comparison, it was also indicated that on average it takes 50 N of force to crack an egg (Bir).

In conclusion, all this information was applied to this experiment. The lever arm angle is the angle of the punch. The numerical values calculated in the study at Wayne State University were used as reference values within this experiment. This research is relevant and can be applied to professional sports, such as boxing and mixed martial arts, as well as self-defense.

Problem Statement

Problem Statement:

To determine how the factors of lever arm, distance of punch applied, and mass of fist will affect the force exerted on a punching bag.

Hypothesis:

If the longest length of the lever arm of 1.4 m is used in combination with the longest distance of punch applied of 0.8 m, and the highest mass of fist of 2.16 kg, then the largest force will be exerted on the punching bag.

Data Measured:

There were three independent variables in this experiment. The first independent variable was the lever arm, with values of 0.8 m, 1.1 m, and 1.4 m. The second independent variable was the distance of the punch applied to the punching bag with values of 0.2 m, 0.5 m, and 0.8 m. The third independent variable was the mass of the fist used to punch the bag with values of 1.94 kg, 2.05 kg, and 2.16 kg. The dependent variable in the experiment was the force exerted on the punching bag, measured in Newtons (N). A Three Factor Design of Experiment (DOE) was used to analyze the data to determine which of the three factors was the most influential in determining the force exerted on the punching bag. There were 5 Three Factor DOEs conducted, resulting in a total of 55 trials.

Experimental Design

Materials:

- Everlast Pro Punching Bag (60 lbs)
- Everlast Pro MMA Grappling Gloves (4 oz.)
- Everlast Pro Boxing Gloves (8 oz.)
- HealthSmart Bathroom Scale (lbs)
- Meter Stick
- TI-Nspire CX Calculator
- iPhone 6 Video Camera
- iPhone 6 Phone Mount
- Toshiba Satellite C55t Laptop
- Screen Protractor V1.1 Software
- Masking Tape

Procedures:

Proper safety precautions should be taken, ensure that the punching bag is securely attached.

1. Set up the punching bag. Stabilize the bunching bag by placing the stabilizing hook where the chain inserts into the ceiling. Refer to Appendix A for instructions on how to set up the punching bag.

2. Step onto the scale and record the weight of the person that will be punching. Convert this weight into kilograms (kg) by dividing the weight by 2.2046.

3. Multiply this mass by 0.0345 (the percentage of the body mass that is accounted for by the fist) in order to calculate the mass of the fist that will be punching the punching bag (low value for mass). Add the mass of the 4oz glove to this mass (standard value for mass), add the mass of the 8oz glove to the low value for mass in order to calculate the high value for mass. Refer to Appendix B for the calculations used in steps 2 – 3.

4. Use the meter stick to measure 0.8 meters from the pivot point, mark this point with the masking tape (low value for lever arm). Use the meter stick to measure 1.1 meters from the pivot point, mark this point with the masking tape (standard value for lever arm). Use the meter stick to measure 1.4 meters from the pivot point, mark this point with the masking tape (high value for lever arm).

5. Use the meter stick to measure 0.2 meters away from the punching bag, mark the ground at this distance with masking tape (low value for distance). Use the meter stick to measure 0.5 meters away from the punching bag in the same direction, mark the ground at this distance with the masking tape (standard value for distance). Use the meter stick to measure 0.8 meters

away from the punching bag in the same direction, mark the ground at this distance (high value for distance). See Figure 2 for a completed setup.

6. Place the iPhone 6 within the iPhone 6 phone mount. Begin recording, then place this phone mount at a distance of 1 meter away from the pivot point.

7. Place Toshiba Satellite C55t Laptop at 3 meters away from the punching bag, in a manner such that the laptop camera is parallel to the punching. Begin recording.

8. Randomize all trials using the TI-Nspire CX random integer function. There are 11 trials in a Three Factor Design of Experiment. Randomize the order of these trials for all 5 DOE's.

9. Refer to the DOE order and conduct the trial accordingly.

10. Conduct each trial 5 times (repeat Step 9 5 times for each DOE trial) such that each trial has 5 sub-trials; this is to ensure more accurate results. The punch should be followed through, such that contact with the bag is applied for as long as possible. The punch should also be relatively quick. Allow the punching bag to oscillate once for each trial, and then stop it. This concludes the physical trials.

11. After all of the trials have been conducted, connect the iPhone 6 to the Toshiba Satellite C55t Laptop and launch the Screen Protractor V1.1 software.

12. Open the video that was recorded with the iPhone 6 and begin playing it. Mark the starting position of the punching bag with one arm using the Screen Protractor.

13. Pause the video when the punching bag is extended the most and begins to return to its starting position. Mark this point with the other arm of using the Screen Protractor. Record the angle that has been calculated and the time elapsed for the punching bag to reach this position from its original starting position. Refer to Appendix B for a sample calculation.

14. Open the video that was recorded with the Toshiba Satellite C55t Laptop. Measure the angle that the punching bag makes with the fist that is punching. Record the angle that has been calculated. Refer to Appendix B for a sample calculation.

15. Convert the angle measured in Step 13 to radians by multiplying by $\frac{\pi}{180°}$, this is necessary in order to calculate angular acceleration (Step 16).

$$\text{Displacement in Radians} = \text{Angle (°) X}\frac{\pi}{180°}$$

16. Use the displacement in radians, calculated in Step 13, and the time elapsed, recorded in Step 12, to calculate the angular acceleration. This equation was

$$Angular\ Acceleration\ (\alpha) = \frac{Displacemnt\ in\ Radians\ (\Delta Ø)}{Change\ in\ Time\ Squared\ (\Delta t^2)}$$

used to calculate angular acceleration. Refer to Appendix B for an example sample calculation.

17. Calculate the moment of inertia of the punching bag, because it is a point mass this is a set value. Refer to Appendix B for a sample calculation.

$$Moment\ of\ Inertia\ (I) = Mass\ X\ Radius\ Squared\ (\mathrm{m\ X}\ r^2)$$

18. Use the moment of inertia, calculated in Step 17, and the angular acceleration, calculated in Step 16, to calculate the torque. Refer to Appendix B for a sample calculation.

$$Torque\ (\boldsymbol{\tau}) = Moment\ of\ Inertia\ X\ Angular\ Acceleration\ (I\ X\ \alpha)$$

19. Use the angle calculated in Step 14, the torque calculated in Step 18, and the lever arm distance to calculate the force of the punch. Refer to Appendix B for a sample calculation.

$$Torque\ (\boldsymbol{\tau}) = Force\ X\ Lever\ Arm\ Distance\ X\ Sine\ of\ the\ \ Angle\ Created\ (F\ X\ l\ X\ \sin(Ø))$$

$$Force\ (F) = \frac{Torque\ (\boldsymbol{\tau})}{Lever\ Arm\ Distance\ X\ Sine\ of\ the\ \ Angle\ Created\ (\ l\ X\ \sin(Ø))}$$

20. Average the forces for the 5 sub – trials, use this average force as the final force for data analysis. Refer to Appendix C for the data from the 5 sub-trials for each trial. Refer to Appendix B for a sample calculation.

21. Repeat steps 12 through 20 for all of the trials. Analyze the data using a Three Factor Design of Experiment.

<u>Diagram</u>:

Figure 1. Materials List

Figure 1 above shows the materials that were used within this experiment. The Everlast Pro Punching Bag (60 lbs.), the Everlast Pro MMA Grappling Gloves (4 oz.), the Everlast Pro Boxing Gloves (8 oz.), the iPhone 6 video camera, the iPhone 6 phone mount, the Toshiba Satellite C55t Laptop preloaded with the Screen Protractor V1.1 Software, the TI-Nspire CX calculator, the masking tape, the bathroom scale, and the meter stick are all shown above. A setup of the bag is shown in Figure 2.

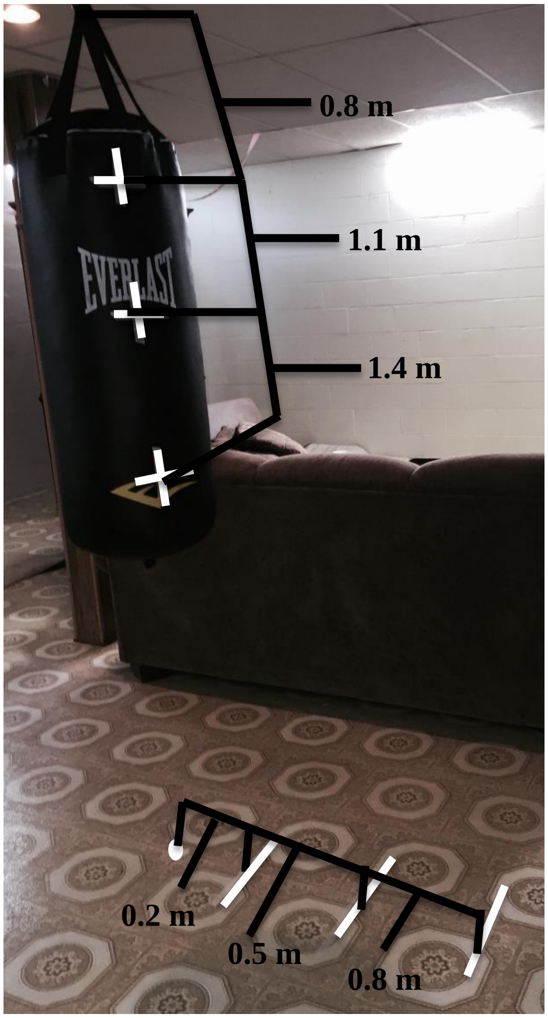

Figure 2. Experimental Setup

Figure 2 above shows how the punching bag was set up for the experiment as well as the marked values for the factors of lever arm and distance. The iPhone was placed in a holder and set up in the ceiling to film the chain of the punching bag. The Toshiba C55-t laptop was set up on a table parallel to the punching bag. Figure 3 shows an image of the chain. The locations marked on the floor represent the distance of the punch; these were the different distances at which the punches were applied. The lengths marked on the bag were the different values for lever arm; these were the different lengths for lever arm and the places where the punches were applied.

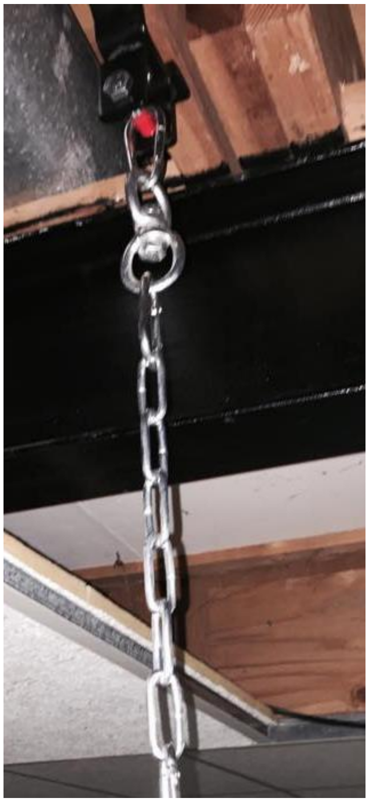

Figure 3. Chain of Punching Bag

Figure 3 shows the chain that was attached to the punching bag. The iPhone was set across from this chain. This chain created was displaced every time the bag was punched. This displacement was the angle that was measured in Step 13.

Data and Observations

Table 1
Factor Values

	Factors		
	Mass (kg)	Lever Arm (m)	Distance (m)
Low	1.94	0.8	0.2
Standard	2.05	1.1	0.5
High	2.16	1.4	0.8

Table 1 above shows the three factors that were used in the experiment, along with the high, standard, and low values for each. The mass of the fist was measured in kg, while the length of the lever arm and the distance from which the punch was applied was measured in m.

Table 2
Data Collected

Force (N)								
Mass	**Lever Arm**	**Distance**	**DOE 1**	**DOE 2**	**DOE 3**	**DOE 4**	**DOE 5**	**Average**
Standard			502.23	591.04	523.37	551.13	533.47	540.25
+	+	+	1356.34	1543.06	1459.50	1597.48	1480.78	1487.43
+	+	-	456.91	484.29	450.85	455.23	493.69	468.19
+	-	+	580.48	575.38	551.65	564.77	569.93	568.44
+	-	-	369.37	395.94	420.41	412.90	401.89	400.10
Standard			639.48	583.78	596.97	511.07	575.72	581.40
-	+	+	792.22	782.87	771.61	775.72	745.82	773.65
-	+	-	376.11	369.89	367.52	387.62	361.12	372.45
-	-	+	325.26	322.52	334.75	299.25	326.68	321.69
-	-	-	167.89	242.67	290.33	287.22	266.47	250.92
Standard			604.89	545.40	618.36	557.13	582.98	581.75

Table 2 above shows the average force of the punches for the 5 DOEs that were conducted. The trials conducted using all high values yielded the highest average force. The trials conducted using all low values yielded the lowest average force. A sample calculation for force can be found in Appendix B.

Table 3
Observations

Trial #	DOE Trial	DOE #	Observations
1	Standard	1	Camera was not recording correctly. Trial redone
3	(+,+,-)	1	Chain wobbled from side to side. Trial redone.
5	(-,-,-)	1	Yielded the lowest force amongst all trials.
6	(-,+,-)	1	Initial punch applied to wrong place on bag. Trial redone.
9	(+,-,-)	1	Motion of chain not completely recorded. Trial redone.
15	(-,-,-)	2	Punching bag was not adjusted properly. Trial redone.
19	(+,-,+)	2	Chain wobbled from side to side. Trial redone.
26	(+,-,-)	3	Camera was not recording correctly. Trial redone
36	(-,-,-)	4	Initial punch applied to wrong place on bag. Trial redone.
40	(+,+,+)	4	Yielded the highest force amongst all trials.
45	Standard	5	Camera turned off during recording. Trial redone.
47	(+,-,-)	5	Wrong values used during trial. Trial was redone.
53	(-,-,+)	5	Camera was not recording correctly. Trial redone
55	Standard	5	Chain wobbled from side to side. Trial redone.

Table 3 shows all significant observations that were noted during data collection.

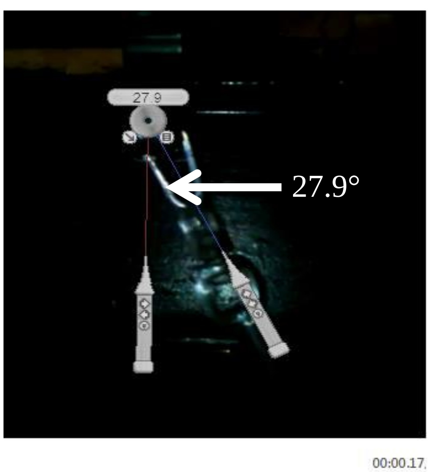

Figure 3. Angle of Displacement

Figure 3 above shows how the angle of displacement was measured from the movement of the chain. The on screen software used to measure the angle of displacement was Screen Protractor V1.1. The angle of displacement was used to calculate the force of the punch applied to the punching bag. The angle of displacement, along with time, was used to calculate the angular acceleration of the punching bag. This angular acceleration was later used to calculate the force of the punch. These formulas were used to calculate the force of the punch.

$$\tau = I\alpha$$

$$\tau = Fl\sin(\text{Ø})$$

Refer to Appendix B for a sample calculation.

Figure 4. Angle of Lever Arm

Figure 4 above shows the lever arm angle that was measured. This angle was measured using the Screen Protractor V1.1. This angle was used to calculate the lever arm. A lever arm sample calculation can be found in Appendix B. This was a (+,+,+) trial. The punch was followed through. This punch resulted in a force of 1,597.48 N. This image demonstrates how the punches were applied. The puncher extends the arm rapidly to make contact with the spot marked on the bag. This punch is followed through, accelerating the bag.

Data Analysis and Interpretation

A Three Factor Design of Experiment (DOE) was conducted to analyze the data. This was the appropriate test to use because there were three independent factors that were apparent within this experiment: the mass, the lever arm, and the distance. The force was the dependent response variable that was being measured. The effect of all of the factors and the interaction between factors was determined. The data is reliable because 5 Three Factor Design of Experiments were conducted to replicate the experiment. This helps eliminate bias that could be present in the data. Each of the trials were randomized using the TI-Nspire CX within the separate DOEs with the standards as the 1^{st}, the 6^{th}, and the 11^{th} trials in the DOEs. The order in which the DOEs were conducted was also randomized. Each trial was independent meaning that one trial did not affect any other trial. The controls within this experiment were the standards. The repletion helps to ensure that the results of the experiment are significant. Randomizing eliminates any experimental error that could occur due to bias.

Table 4
Standard Values for Force

Force (N)				
502.23	591.04	523.37	551.13	533.47
639.48	583.78	596.97	511.07	575.72
604.89	545.40	618.36	557.13	582.98

Table 4 above shows the standard values for all 5 DOEs. The range of the standards is 137.24 N. This value was derived by calculating the difference between the maximum standard value, 639.48 N, and the minimum standard value, 502.23 N.

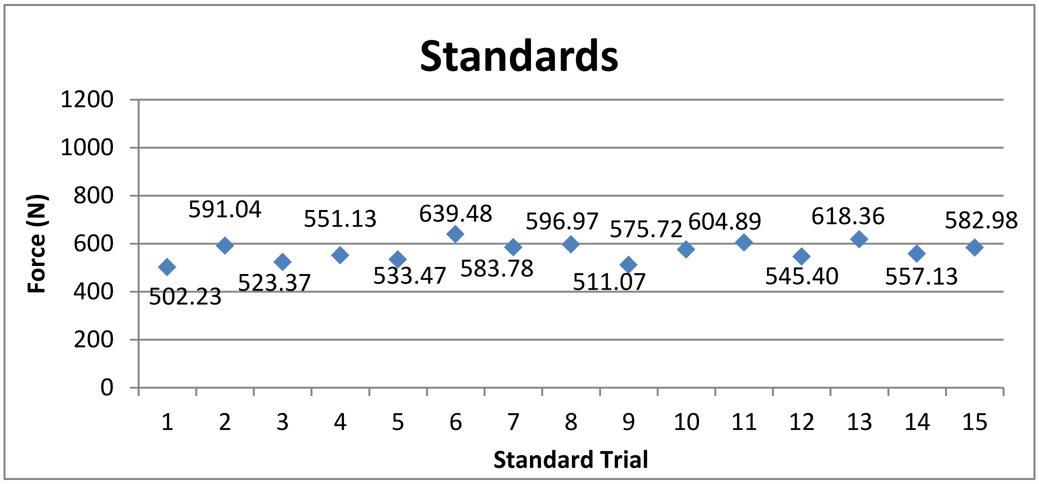

Figure 5. Scatterplot of Standards

Figure 5 above shows a scatter plot of the standards from all five DOE's. There is no noticeable trend within the data and the variability within the standards is miniscule compared to the rest of the data. The range of the standards is 137.24 N.

Table 5
Average Results from DOE

Mass	Lever Arm	Distance	Force (N)
Standard	Standard	Standard	540.25
+	+	+	1487.43
+	+	-	468.19
+	-	+	568.44
+	-	-	400.10
Standard	Standard	Standard	581.40
-	+	+	773.65
-	+	-	372.45
-	-	+	321.69
-	-	-	250.92
Standard	Standard	Standard	581.75

Table 5 above has the average results from all five DOEs. This data was used to conduct a statistical analysis test, a Three Factor Design of Experiment. The grand average was calculated to be 580.36 N.

Table 6
Effect of Mass

(-) Values (N)	(+) Values (N)
773.65	1487.43
372.45	468.19
321.69	568.44
250.92	400.10
Average: 429.68	Average: 731.04

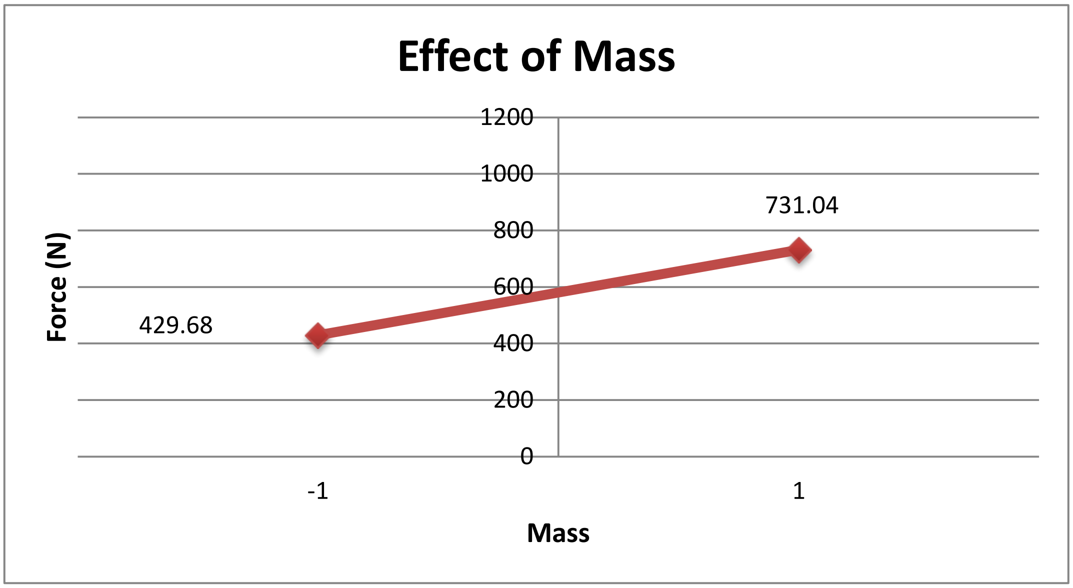

Figure 6. The Effect of Mass

Table 6 shows the effect of the mass. As the mass of the fist increases, the force of the punch also increases. The effect of the mass of the fist is 301.36 N. This effect is the difference between the high and the low averages for mass. Figure 6 graphically shows the effect of the mass of the fist. As the mass of the fist increases, so does the force of the punch. On average, as the mass of the fist goes from low to high, the force exerted by the punch increases by 301.36 N.

Table 7
Effect of Lever Arm

(-) Values (N)	(+) Values (N)
568.44	1487.43
400.10	468.19
321.69	773.65
250.92	372.45
Average: 385.29	Average: 775.43

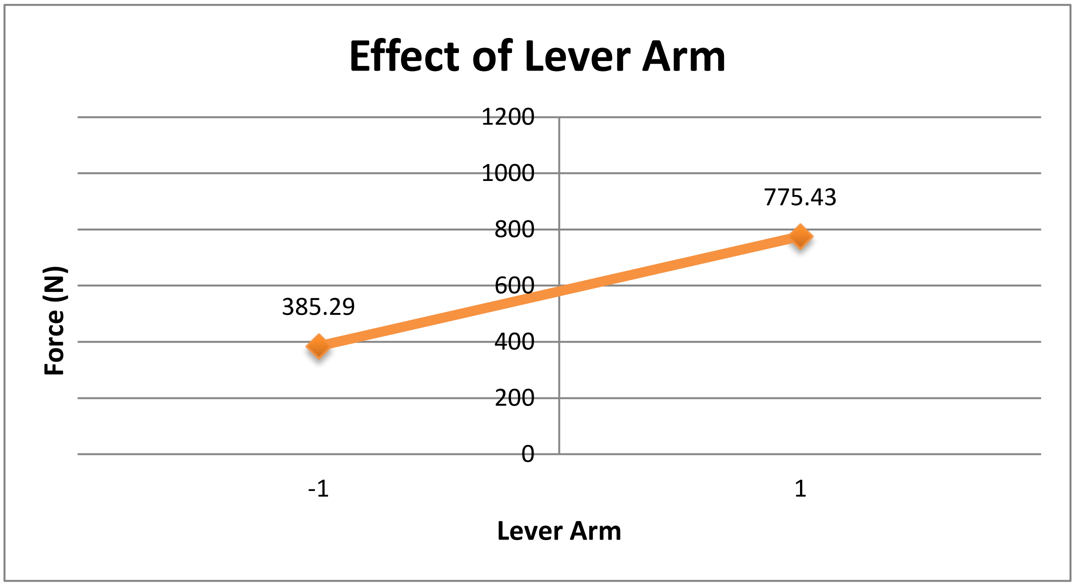

Figure 7. The Effect of Lever Arm

Table 7 shows the effect of the lever arm. As the lever arm increases, the force of the punch also increases. The effect of the lever arm is 390.14 N. This effect is the difference between the high and the low averages for the lever arm. Figure 7 graphically shows the effect of lever arm. As the lever arm increases, so does the force of the punch. On average, as the lever arm goes from low to high, the force exerted by the punch increases by 390.14 N.

Table 8
Effect of Distance

(-) Values (N)	(+) Values (N)
468.19	1487.43
400.10	568.44
372.45	773.65
250.92	321.69
Average: 372.92	Average: 787.80

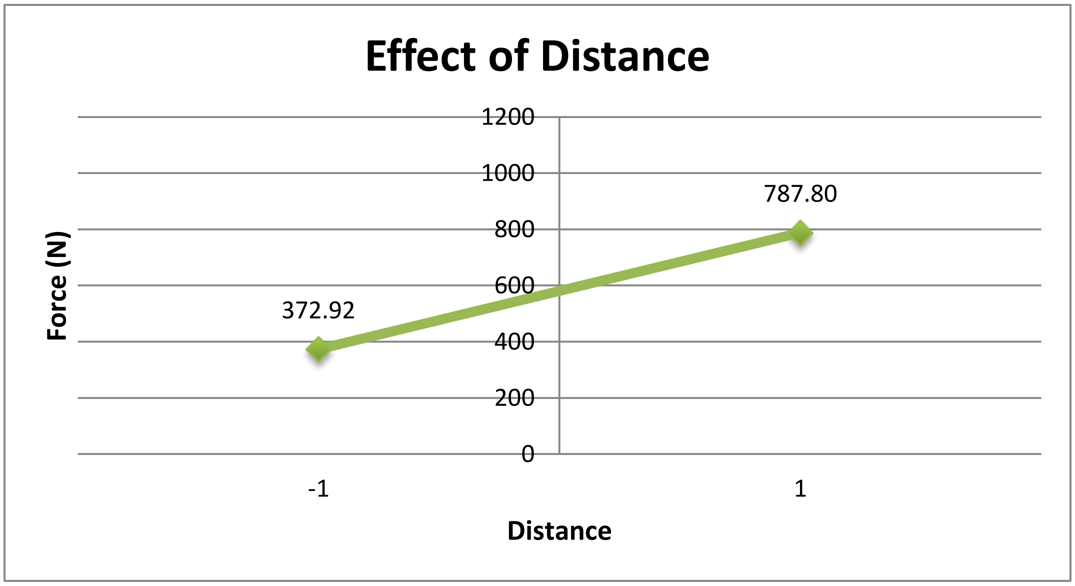

Figure 8. The Effect of Distance

Table 8 shows the effect of the distance at which the punch was applied. As distance increases, the force of the punch also increases. The effect of distance is 414.89 N. This effect is the difference between the high and the low averages for the distance. Figure 8 graphically shows the effect of distance. As the distance increases, so does the force of the punch. On average, as the distance goes from low to high, the force exerted by the punch increases by 414.89 N.

Table 9
Interaction Effect of Mass and Lever Arm

	Lever Arm (-) (N)	Lever Arm (+) (N)
Line segment (+) (solid) Mass	484.27	977.81
Line segment (-) (dashed) Mass	286.31	573.05

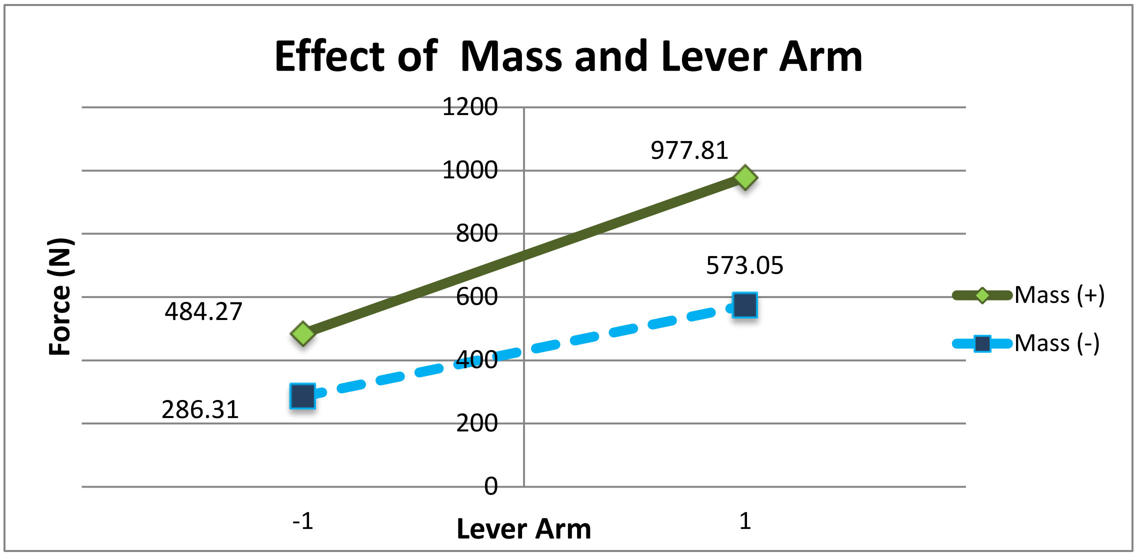

Figure 9. The Interaction Effect of Mass and Lever Arm

Table 9 shows the values for when both mass and the lever arm were held high and low. Figure 9 shows the interaction between the mass of the fist and the lever arm. The solid segment shows when the lever arm goes from low to high while mass is held high. The slope of the solid segment is 246.77 N. The dashed segment shows when the lever arm goes from low to high while mass is held low. The slope of the dashed segment is 143.37 N. The effect of the interaction of mass and the lever arm is 103.40 N or the difference between the two slopes. The force exerted by the punch increases by different amounts as the lever arm goes from low to high while mass is held both high and low. This suggests that there may be an interaction between mass of the fist and the lever arm. The slopes of the lines are not equal, therefore, there is a possibility of an interaction.

Table 10
Interaction Effect of Mass and Distance

	Distance (-) (N)	Distance (+) (N)
Line segment (+) (solid) Mass	434.15	1027.94
Line segment (-) (dashed) Mass	311.69	547.67

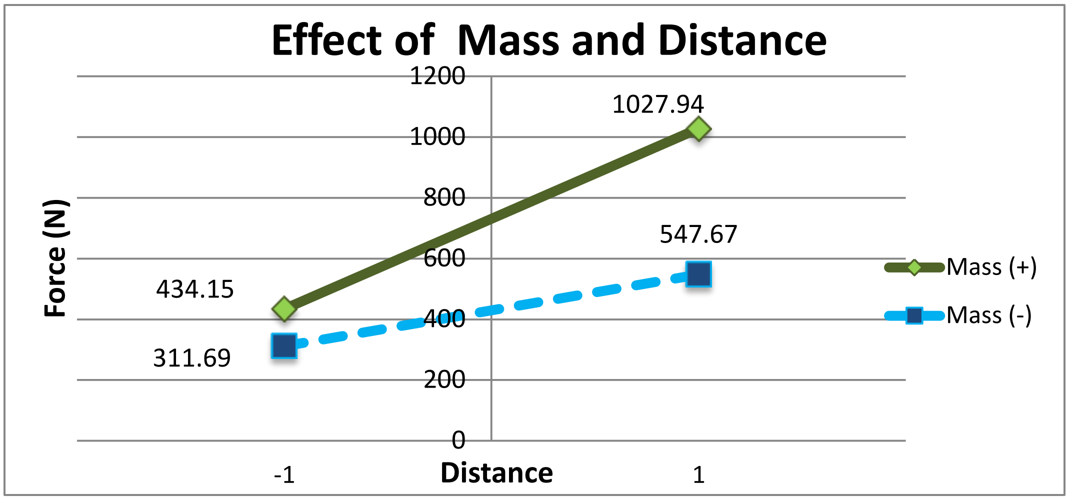

Figure 10. The Interaction Effect of Mass and Distance

Table 10 shows the values for when both mass and distance were held high and low. Figure 10 graphically shows the interaction between the mass of the fist and the distance at which the punch was applied. The solid segment shows when the distance goes from low to high while mass is held high. The slope of the solid segment is 296.89 N. The dashed segment shows when the distance goes from low to high while mass is held low. The slope of the dashed segment is 117.99 N. The effect of the interaction of mass and distance is 178.90 N. The force exerted by the punch increases by different amounts as distance goes from low to high while mass is held both high and low, suggesting that there may be an interaction between the mass of the fist and distance of the punch.

Table 11
Interaction Lever Arm and Distance

	Distance (-) (N)	Distance (+) (N)
Line segment (+) (solid) Lever Arm	420.32	1,130.54
Line segment (-) (dashed) Lever Arm	325.51	445.07

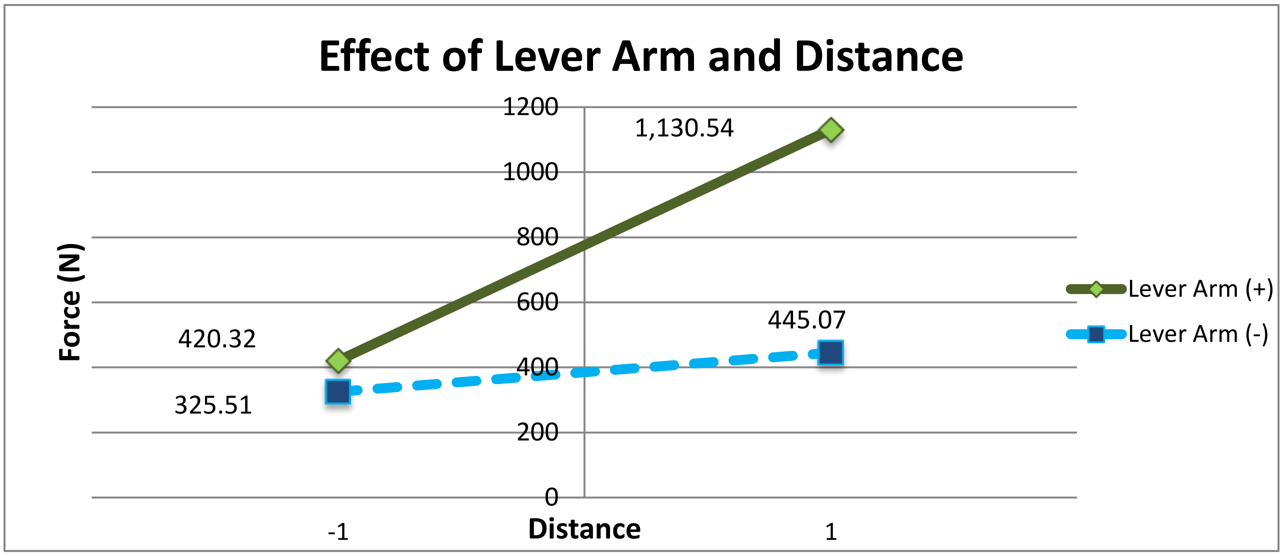

Figure 11. The Interaction Effect of Lever Arm and Distance

Table 11 shows the values for when both lever arm and the distance were held high and low. Figure 11 graphically shows the interaction between the lever arm and the distance at which the punch was applied. The solid segment shows when the distance goes from low to high while the lever arm is held high. The slope of the solid segment is 355.11 N. The dashed segment shows when the distance goes from low to high while the lever arm is held low. The slope of the dashed segment is 59.78 N. The effect of the interaction of the lever arm and distance is 295.33 N. The force exerted by the punch increases by different amounts as the distance goes from low to high while the lever arm is held both high and low. This suggests that there may be an interaction between the lever arm and the distance at which the punch is applied.

Table 12
Effect of All Factors and Interactions

Effect	Force (N)
Mass (M)	301.36
Lever Arm (L)	390.14
Distance (D)	414.89
Mass and Distance (MD)	178.90
Mass and Lever Arm (ML)	103.40
Lever Arm and Distance (LD)	295.33

Table 12 shows all of the effects and the variables that correspond with them. These effects are plotted in Figure 12 in order to determine which factors are significant and which are not.

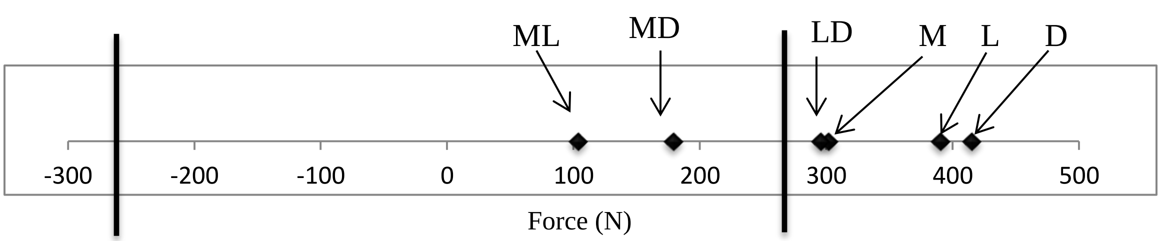

Figure 12. Significance Test

Figure 12 shows the significance test that was conducted to determine which factors and combination of factors were statistically significant. The significance level was twice the range of the standards. As shown in Table 4 and Figure 5, the range of the standards was 137.24 N. Two times this ranges is 274.49 N. Any value outside of this range is significant. This range is graphed with deviation bars 274.49 N away from 0 in both the positive and negative direction. According to this test, the interaction effect of mass and distance, and the interaction of mass and lever were statistically insignificant. The factor that had the greatest effect upon the force exerted by the punch was the distance at which the punch was applied. The second most significant factor was the lever arm, followed by the mass, and then the interaction between the lever arm and the distance at which the punch was applied. The prediction equation and parsimonious prediction equation are both shown with sample calculations in Appendix B.

Conclusion

The purpose of this experiment was to determine which combination of the three factors of lever arm, distance at which a punch is applied, and the mass of the fist used to apply the punch, produced the strongest punch as well as which of the three factors had the largest effect on the force of the punch. The lever arm and distance were manipulated using different measurements, while the mass of the fist was manipulated using MMA ProFighter gloves. The different lengths of the lever arm were 0.8 m, 1.1 m, and 1.4 m. The different distances at which the punch was applied to the punching bag were 0.2 m, 0.5 m, and 0.8 m. The different masses that were used to punch the bag were 1.94 kg, 2.05 kg, and 2.16 kg. The hypothesis stating that the longest length of the lever arm combined with the longest distance of punch applied and the highest mass of fist would produce the most force was accepted. The trials that used a lever arm of 1.4 m, a distance of 0.8 m and a mass of fist of 2.16 kg produced 1487.43 N of force, on average. The data necessary for this experiment was collected using video recordings of the punches applied to a punching bag. These recordings were then analyzed to calculate the angular acceleration. This, alongside moment of inertia and torque, was used to calculate the force of the punch. The data was further analyzed by conducting 5 Three Factor Design of Experiments (DOEs) in order to determine the significance of the factors.

The results of the DOEs concluded that the factor that had the greatest effect upon the force exerted by the punch was the distance at which the punch was applied. The second most significant factor was the lever arm, followed by the mass of the fist, and then the interaction between the lever arm and the distance at which the punch was applied.

The most significant factor was the distance at which the punch was applied, with an effect of 414.89 N. This means that as the value of distance increased, from low to high, the force exerted by the punch increased as well. This is supported by Newton's Second Law of Motion, which states that an unbalanced force causes an object to accelerate. This law can be summarized using the equation F = ma, where m represents the mass of the object, a represents the acceleration of the object, and F represents the force exerted by the object. This is also explained through the equation $Ft = m\Delta v$. This equation calculates the momentum of an object. The faster an object goes, the more momentum is has. Likewise the faster an object goes the more force at has. The distance at which the punch was applied had the greatest effect because the longer the distance, the faster the fist was able to accelerate and gain speed in order to produce a more forceful punch. This force was then transferred from the fist to the punching bag through contact. The momentum of the punch was transferred to the punching bag. This transfer of momentum also results in a transfer of force.

The second most significant factor was lever arm, with an effect value of 390.14 N. This means that as lever arm increased, from low to high, the force of the punch increased as well. As the length of the lever arm increased, the angular displacement of the punching bag also increased. The increased angular displacement resulted in an increased angular acceleration which led to a larger calculated force. As lever arm gets longer, it is easier to rotate an object. The lever arm was the length that the punch was applied at. This length of the lever arm was the length from the axis of rotation to a marked point on the punching bag.

The third most significant factor was the mass of the fist, with an effect value of 301.36 N. This means that as the mass of the fist increased, from low to high, the force of the punch increased as well. This is explained through Newton's Second Law of Motion. As stated earlier, this law can be summarized using the equation $F = ma$. As the mass of the object (in this case, the fist) increases, the force exerted by the punch also increases.

The interaction between the lever arm and the distance at which the punch was applied was the fourth most significant factor, as well as the only significant interaction effect. The effect value was 295.33 N. This means that as the lever arm and the distance at which the punch was applied increased, the force of the punch increased as well. Since the lever arm and the distance are both significant factors that increase the force of the punch, it is only reasonable that a combination of both factors would increase the force of a punch as well.

The interaction between the mass of the fist and the lever arm had the smallest effect on the force of the punch. The effect value for this interaction was 103.40 N. This means that as both the mass of the fist and the lever arm increased, so did the force of the punch. However, this increase was not as significant as the other factors.

The results of this experiment agree with previous work that was done in the field of sports science and kinesiology. As researched, heavyweight boxers and MMA fighters are able to produce more forceful punches compared to lighter athletes (Cheraghi). This corresponds with the significance of the mass of the fist in relation to the force of a punch. As mass of the fist increased, so did the force. Professional boxers and MMA fighters also deliver punches from a certain distance, not only to avoid blows from their opponent, but to maximize the force of their own blows as well. This corresponds with the significance of the distance at which the punch was applied. When the value for distance was high, the force was greatest. Professional fighters also seek to land key blows to the head. This is because with the human body, the feet serve as the axis of rotation, and the head is the longest possible length for lever arm. This means that if a punch is landed on higher up on a person's body they will fall to the ground more easily, a knockout in professional sporting. This corresponds with the significance of the lever arm in relation to the force of a punch. As the lever arm increases, the force of the punch increases as well.

No experiment is perfect, and errors within an experiment can cause confounding and bias within the data. One weakness in the design of the experiment was poor lighting. Poor lighting was the cause of dark footage that was difficult to analyze. Another error in this experiment was inaccuracy. The punches were not applied to the exact same area of the punching bag every single time. This was accounted for by running numerous trials with the same factor values and then averaging the forces to eliminate any extreme values present in the data.

There are many different ways to further this research. Different types of punches, such as hooks and uppercuts can be analyzed to determine the strongest type of punch. Also, better video analytical software can be used. The software used to analyze the videos was Windows Moviemaker. This software had a frame rate of 60 fps. Using more precise video software would result in more accurate data. The effect of a punch on moving targets could be analyzed as well. The energy of a punch or the momentum of a punch can also be researched. This research could be applied to professional sports such as boxing and Mixed Martial Arts. It could also be applied to basic self-defense. Regular people can wear gloves when they are walking around. Having gloves on increases the mass of the fist, resulting in a more forceful punch. Fighters can punch their opponents at different lengths, from different distances, with different masses of gloves to maximize the force of their punches.

Appendix A: Punching Bag Setup

Materials:

Everlast Pro Punching Bag (60 lbs)
Husky 3/8 in. Roundhead Ratchet
DEWALT Titanium Pilot Point Drill Bit Set
Zircon Corporation StudSensor
BLACK &DECKER 20-Volt Max Power Drill
Husky 1/2 in. Socket

Procedures:

Proper safety precautions should be taken, ensure that the punching bag is securely attached. Caution should be taken when handling the power drill.

1. Locate a piece of wood within the ceiling. Use the stud detector to accomplish this task.

2. Open the box containing the punching bag and place the wood beam hanger onto the wood. Mark the points to screw in the hanger.

3. Use the power drill and the ¼ in. drill bit to drill two holes for the screws. The holes should be approximately 1 in. in depth.

4. Use the socket wrench and the ½ in socket to secure the wood beam hanger into the holes made in step 3.

5. Attach the chain for the punching bag onto the wood beam hanger. The end result of steps 1 to 5 is shown in Figure 2, below.

6. Secure the hook on the punching bag to the chain that is now being suspended from the ceiling. Make sure the bag is secure by moving it a little. The finished punching bag set up is shown in Figure 3, below.

Diagram:

Figure 1. Materials for Setting up Punching Bag

Figure 1 above shows the materials that were used to set up the punching bag. These materials include the power drill, the socket wrench, the socket, the drill bits, the punching bag, and the stud detector.

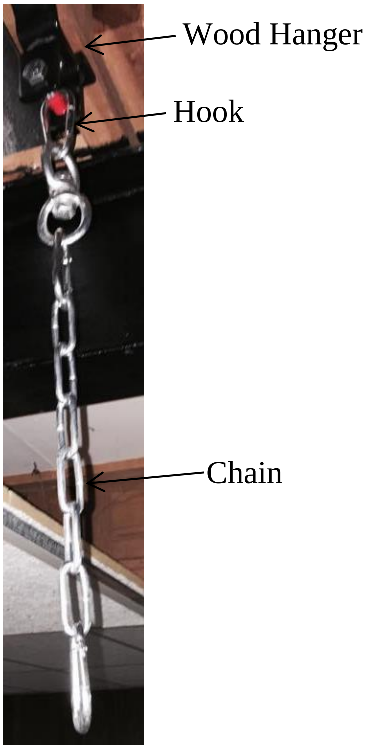

Figure 2. Chain Setup

Figure 2 above shows the finished chain setup. The chain is hooked on to the wood hanger. This is the end result of steps 1 through 5.

Figure 3. Finished Bag Setup

Figure 3 above shows the final setup of the bag. The bag is now hooked on to the chain.

Appendix B: Formulas and Sample Calculations

Mass of the Fist Used to Execute the Punch

One of the three factors tested in this experiment was the mass of the fist used to punch the punching bag. An average human fist is 3.45% of the total body weight. Therefore, mass of the fist is equal to the body weight multiplied by 0.0345. Also, for the trials that used the standard or high values for mass of the fist, the mass of the particular glove used was added as well. This is shown in the equation below.

$$Mass\ of\ Fist = (Body\ Weight \times 0.0345) + Mass\ of\ Fighting\ GLove$$

In the equation above, mass of the fist, body weight, and mass of the fighting glove are all measured in kg. A sample calculation of the mass of the fist used to deliver the punch is shown below.

$$Mass\ of\ Fist = (Body\ Weight \times 0.0345) + Mass\ of\ Fighting\ Glove$$

$$Mass\ of\ Fist = (124\ kg \times 0.0345) + 2.16\ kg$$

$$Mass\ of\ Fist = 6.44\ kg$$

Figure 1. Sample Calculation of Mass of Fist

Figure 1 above shows a sample calculation of the mass of the fist used to deliver a punch using the heavy fighting glove.

Angular Acceleration

Angular acceleration is the change in angular velocity that a spinning object undergoes per unit time. When the velocity of an object is not constant, the angular acceleration is equal to the rotational displacement per unit time squared. The formula for the angular acceleration of the punching bag is shown below.

$$\alpha = \frac{\theta}{t^2}$$

The angle of displacement of the punching bag is represented by the symbol θ. Time is represented by t. The angle of displacement was measured in degrees and then converted into radians. Time was measured in seconds while angular acceleration is measured in rad/s^2. A sample calculation of the angular acceleration of the punching bag is shown below.

$$\alpha = \frac{\theta}{t^2}$$

$$\alpha = \frac{0.637\,rad}{(0.195\,s)^2}$$

$$\alpha = 16.75\,rad/s^2$$

Figure 2. Sample Calculation of Angular Acceleration

Figure 2 above shows a sample calculation of the angular acceleration of the punching bag. The values for angle of displacement and time were taken from the first standard trial of DOE 1.

Moment of Inertia

The moment of inertia is the same as rotational inertia, or the resistance of an object to the motion of rotation. It is also known as the rotational mass. Moment of Inertia is dependent on the shape of the object as well as the axis of rotation. The rotation of the punching bag resembles a point mass, as the bag is hanging from the ceiling. The formula used to calculate the moment of inertia of the punching bag is shown below

$$I = \mathrm{m}r^2$$

In the equation above, the moment of inertia is denoted by the symbol I and is measured in kg*m^2. Mass of the punching bag is denoted by the symbol m and measured in kg, while the radius of the bag, from the top of the chain to the center of the punching bag, is denoted by the symbol r and measured in meters. A sample calculation of the moment of inertia of the punching bag is shown below.

$I = \mathrm{m}r^2$

$I = 27.22\ \mathrm{kg}\ \times (1.1\ m)^2$

$I = 32.93\ \mathrm{kg} \times m^2$

Figure 3. Sample Calculation of Moment of Inertia of Punching Bag

Figure 3 above shows a sample calculation of the moment of inertia of the punching bag. The moment of inertia was constant, as the same punching bag was used throughout the experiment.

Torque

Torque is a force that causes an object to rotate. In this experiment, the punching bag was rotated by the force of the punch that was applied to the bag. Torque is the product of the moment of inertia and angular acceleration. The equation used to calculate the torque applied to the punching bag is shown below.

$$\tau = I\alpha$$

In the equation above, torque is denoted by the symbol τ and is measured in Newtons. The moment of inertia is denoted by the symbol I and is measured in kg*m^2. Angular acceleration is denoted by the symbol α and is measured in rad/s^2. A sample calculation for the torque exerted on the punching bag is shown below.

$\tau = I\alpha$

$\tau = (32.93\ \text{kg} \times m^2)(\ 16.75\ rad/s^2)$

$\tau = 551.58\ N$

Figure 4. Sample Calculation for the Torque Applied to Punching Bag

Figure 4 above shows a sample calculation for the torque applied to the punching bag by the punch. The moment of inertia was constant for every trial while the value for the angular acceleration was taken from the first standard trial of DOE 1.

Force

Force can be calculated if torque, the length of the lever arm, and the angle at which torque is being applied is known. The equation used to calculate the force exerted by the punch on the punching bag is shown below.

$$F = \frac{\tau}{(L \times \sin\theta)}$$

In the equation shown above, force is represented by the symbol F while torque represented by the symbol τ. Both are measured in Newtons. The length of the lever arm is represented by the symbol L and is measured in meters. The symbol θ, in this equation, represents the angle at which torque is applied to the punching bag. It is measured in radians. A sample calculation of force is shown below.

$$F = \frac{\tau}{(L \times \sin\theta)}$$

$$F = \frac{551.58\ N}{(1.1\ \text{m})\ \sin(1.62)}$$

$$F = 502.13\ N$$

Figure 5. Sample Calculation for Force Exerted on the Punching Bag

Figure 5 above shows a sample calculation for the force exerted on the punching bag by the punch. The values for length of the lever arm and angle of force application were taken from the first standard trial of DOE 1.

DOE Prediction Equations

A key part of the Design of Experiment (DOE) statistical analysis is the prediction equation. The prediction equation is derived by using the grand average as well as the average effects of the factors and interactions. The prediction equation can be used to determine the force of any given trial by designating whether or not the factor was held high or low for that particular trial. The prediction equation is shown below.

$$y = 580.36 + \frac{301.36}{2}M + \frac{390.14}{2}L + \frac{414.89}{2}D + \frac{178.90}{2}MD + \frac{103.40}{2}ML + \frac{295.33}{2}LD + Noise$$

In the equation shown above, mass of the fist is denoted by M, lever arm is denoted by L, and the distance at which the punch is applied is denoted by D. The pairs of variables represent the different interactions of the three factors. Noise takes into account any lurking variables which were not accounted for during the experiment. In order to designate whether a particular factor was held high or low, either 1 or -1 must be substituted into the equation. A sample calculation is shown below.

$$y = 580.36 + \frac{301.36}{2}M + \frac{390.14}{2}L + \frac{414.89}{2}D + \frac{178.90}{2}MD + \frac{103.40}{2}ML + \frac{295.33}{2}LD + Noise$$

$$y = 580.36 + \frac{301.36}{2}(1) + \frac{390.14}{2}(1) + \frac{414.89}{2}(1) + \frac{178.90}{2}(1)(1) + \frac{103.40}{2}(1)(1) + \frac{295.33}{2}(1)(1) + Noise$$

$$y = 1422.37\ N$$

Figure 6. Sample Calculation Using the Prediction Equation

Figure 6 above shows a sample calculation of a trial using the prediction equation. The trial used was the trial in which all the factors were held high. Therefore, 1 was substituted for every variable and interaction.

However, the prediction equation is redundant and inaccurate due to accounting for insignificant factors and interaction. Using a mathematical significance test determined which factors and interactions were significant. Afterwards, a smaller, more accurate equation was formed. This is known as the parsimonious prediction equation.

$$y = 580.36 + \frac{301.36}{2}M + \frac{390.14}{2}L + \frac{414.89}{2}D + \frac{295.33}{2}LD + Noise$$

Notice in the equation above, the interactions between the mass of the fist used to execute the punch and distance as well as the interaction between the mass of the fist and lever arm were excluded. This was due to the interactions being insignificant. The factors of mass of fist, lever arm, and the distance at which the punch was applied as well as the interaction between lever arm and distance were the only variables that were significant. A sample calculation using the parsimonious prediction equation is shown below.

$$y = 580.36 + \frac{301.36}{2}M + \frac{390.14}{2}L + \frac{414.89}{2}D + \frac{295.33}{2}LD + Noise$$

$$y = 580.36 + \frac{301.36}{2}(1) + \frac{390.14}{2}(1) + \frac{414.89}{2}(1) + \frac{295.33}{2}(1)(1) + Noise$$

$$y = 1281.2\ N$$

Figure 7. Sample Calculation Using the Parsimonious Prediction

Figure 7 above shows a sample calculation using the parsimonious prediction equation. The trial calculated was the trial in which all three factors were held high. Therefore, 1 was substituted for the variables and interactions.

Appendix C: Raw Data Tables

Table 1
Raw Data Table

Trial	Degrees (°)	Time (s)	Angular Acceleration (m/s^2)	Torque (N*m)	Angle lever arm (Radians)	Lever Arm Distance (M)	Force (N)
S1.1	36.50	0.195	16.75	551.70	1.62	1.1	502.23
S1.2	36.65	0.180	19.74	650.14	1.57	1.1	591.04
S1.3	36.16	0.190	17.48	575.71	1.57	1.1	523.37
S1.4	36.10	0.185	18.41	606.24	1.58	1.1	551.13
S1.5	35.70	0.187	17.82	586.76	1.56	1.1	533.47
S1							**540.25**
S2.1	35.37	0.170	21.36	703.42	1.57	1.1	639.48
S2.2	36.20	0.180	19.50	642.16	1.57	1.1	583.78
S2.3	34.99	0.175	19.94	656.67	1.57	1.1	596.97
S2.4	35.31	0.190	17.07	562.17	1.57	1.1	511.07
S2.5	35.70	0.180	19.23	633.29	1.57	1.1	575.72
S2							**581.40**
S3.1	35.05	0.174	20.21	665.38	1.57	1.1	604.89
S3.2	35.34	0.184	18.22	599.94	1.57	1.1	545.40
S3.3	35.42	0.173	20.66	680.20	1.57	1.1	618.36
S3.4	36.10	0.184	18.61	612.84	1.57	1.1	557.13
S.3.5	35.75	0.179	19.47	641.28	1.57	1.1	582.98
S3							**581.75**
HHH.1	45.01	0.120	54.55	1796.49	1.24	1.4	1356.34
HHH.2	42.18	0.110	60.84	2003.55	1.19	1.4	1543.06
HHH.3	44.69	0.116	57.97	1908.86	1.21	1.4	1459.50
HHH.4	44.30	0.110	63.90	2104.25	1.23	1.4	1597.48
HHH.5	43.60	0.113	59.59	1962.50	1.24	1.4	1480.78
HHH							**1487.43**
HHL.1	27.90	0.170	16.85	554.86	1.05	1.4	456.91
HHL.2	26.40	0.160	18.00	592.71	1.06	1.4	484.29
HHL.3	25.30	0.156	18.14	597.52	1.24	1.4	450.85
HHL.4	27.89	0.163	18.32	603.33	1.24	1.4	455.23
HHL.5	28.06	0.157	19.87	654.29	1.24	1.4	493.69
HHL							**468.19**
HLH.1	45.70	0.250	12.76	420.26	1.13	0.8	580.48
HLH.2	45.35	0.250	12.66	417.04	1.13	0.8	575.38
HLH.3	40.70	0.240	12.33	406.12	1.17	0.8	551.65

HLH.4	42.80	0.245	12.44	409.82	1.14	0.8	564.77
HLH.5	46.09	0.254	12.47	410.60	1.12	0.8	569.93
HLH							**568.44**
HLL.1	39.34	0.280	8.76	288.40	1.35	0.8	369.37
HLL.2	38.30	0.270	9.17	301.96	1.26	0.8	395.94
HLL.3	37.60	0.260	9.71	319.68	1.25	0.8	420.41
HLL.4	39.42	0.270	9.44	310.79	1.23	0.8	412.90
HLL.5	38.02	0.270	9.10	299.75	1.20	0.8	401.89
HLL							**400.10**
LHH.1	45.90	0.170	27.72	912.84	0.97	1.4	792.22
LHH.2	45.60	0.169	27.87	917.64	0.99	1.4	782.87
LHH.3	45.08	0.168	27.91	919.10	1.02	1.4	771.61
LHH.4	45.87	0.170	27.70	912.24	1.00	1.4	775.72
LHH.5	44.30	0.170	26.75	881.02	1.00	1.4	745.82
LHH							**773.65**
LHL.1	44.03	0.230	14.53	478.38	1.14	1.4	376.11
LHL.2	46.18	0.235	14.59	480.61	1.19	1.4	369.89
LHL.3	47.12	0.239	14.40	474.12	1.17	1.4	367.52
LHL.4	48.12	0.237	14.95	492.39	1.14	1.4	387.62
LHL.5	44.45	0.238	13.70	451.02	1.10	1.4	361.12
LHL							**372.45**
LLH.1	39.60	0.320	6.75	222.27	1.02	0.8	325.26
LLH.2	44.04	0.330	7.06	232.43	1.12	0.8	322.52
LLH.3	44.72	0.325	7.39	243.34	1.14	0.8	334.75
LLH.4	41.88	0.330	6.71	221.03	1.18	0.8	299.25
LLH.5	44.35	0.330	7.11	234.07	1.11	0.8	326.68
LLH							**321.69**
LLL.1	31.28	0.400	3.41	112.36	0.99	0.8	167.89
LLL.2	30.69	0.340	4.63	152.59	0.90	0.8	242.67
LLL.3	28.97	0.300	5.62	185.01	0.92	0.8	290.33
LLL.4	31.20	0.310	5.67	186.60	0.95	0.8	287.22
LLL.5	31.15	0.320	5.31	174.84	0.96	0.8	266.47
LLL							**250.92**

The figure above shows the raw data tables that were used. The data tables are labeled with L for low, S standard, and H for high.

Works Cited

Benson, Tom. "Angular Displacement, Velocity, Acceleration." *NASA.gov*. National Aeronautics and Space Administration, 09 Sept. 2014. Web. 24 Mar. 2015. <http://www.grc.nasa.gov/WWW/k-12/airplane/angdva.html>.

Bir, Cindy. "Brute Force: Humans Can Sure Take a Punch." *LiveScience.* TechMedia Network, 03 Feb. 2010. Web. 17 Apr. 2015. <http://www.livescience.com/6040-brute-force-humans-punch.html>.

Cheraghi, Mahdi, Hamid Alinejad, and Ahmad Arshi. "Kinematics of a Straight Right Punch in Boxing." *Citefactor.org*. Annals of Applied Sport Science Vol 2 no 2, pp 39-50, 1 Aug. 2014. Web. 24 Mar. 2015. <http://www.citefactor.org/article/index/12615/pdf/kinematics-of-straight-right-punch-in-boxing#.VRIK__nF-So>.

Fitzpatrick, Richard. "Torque." University of Texas at Austin, 02 Feb. 2006. Web. 23 Mar. 2015. <http://farside.ph.utexas.edu/teaching/301/lectures/node104.html>.

Nave, Rod. "Basic Rotational Quantities." *Rotational Quantities*. Georgia State University, 6 Nov. 2014. Web. 20 Mar. 2015. <http://hyperphysics.phy-astr.gsu.edu/hbase/rotq.html>.

Richmond, Michael. "Physics & Biomechanics Glossary: Angular Velocity." *Torque and Rotational Equilibrium*. Rochester Institute of Technology, 15 Apr. 2014. Web. 22 Mar. 2015. <http://spiff.rit.edu/classes/phys211/lectures/torq/torq_all.html>.

Vawter, Richard. "The Rotational Force." Western Washington University, 4 Apr. 2010. Web. 24 Mar. 2015. <http://faculty.wwu.edu/vawter/PhysicsNet/Topics/RotationalDynamics/Torque.html>